MATHEMATICS HSC HALF YEARLY EXAM 2006

QUESTION 1 **15 marks** Start a NEW answer booklet **Marks**

(a) Calculate, correct to 3 significant figures, the value of: **2**

$$\frac{\sqrt{(2.3)^5 - 6.95}}{\pi}$$

(b) Solve the equation $p^2 = 2p$ **2**

(c) Fully factorise the expression: $5x^3 + 40$ **2**

(d) Briefly explain why the equation $|5t - 32| = -18$ has no solutions. **1**

(e) Find the two values of x where the circle $x^2 + y^2 = 15$ and the line $y = 2x$ intersect. Leave your answers in exact form. **3**

(f) (i) Solve the quadratic inequality $36 - 9x - x^2 \geq 0$. **2**

(ii) Graph your solution on a number line. **1**

(g) Solve for m: **2**

$$7^m = 18$$

Give your answer correct to 2 decimal places.

QUESTION 2 **15 marks** Start a NEW answer booklet **Marks**

The points P and Q have coordinates (3, –2) and (1, 3) respectively.

(a) The line k has equation $4x + 5y - 2 = 0$. Verify that P lies on k. **1**

(b) The line l through Q has gradient $\frac{1}{3}$. Show that the equation of l is **2**

$$x - 3y + 8 = 0.$$

(c) The point of intersection of k and l is R. Find the coordinates of R. **3**

(d) Draw a neat sketch on a number plane showing P, Q, R, k and l. **2**

(e) Find the perpendicular distance of P from l. **2**
Leave your answer as a surd.

(f) Find the area of ΔPQR. **3**

(g) l is a tangent to a circle centre P. Find the equation of that circle. **2**

QUESTION 3 **15 marks** Start a NEW answer booklet **Marks**

(a) Solve the equation $3x^2 - 6x + 1 = 0$ giving each solution correct to two decimal places. **2**

(b) Let α and β be the roots of the equation $x^2 - 5x + 2 = 0$.
Find the values of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha + 1)(\beta + 1)$ **2**

(c) Consider the equation $x^2 + (k+2)x + 4 = 0$.

For what values of k does the equation have:

(i) equal roots? **2**

(ii) distinct real roots? **1**

(d) Solve the equation $3^{2x} + 2 \times 3^x - 15 = 0$ **3**

(e) Find the values of *A* and *B* for which the following identity holds. **3**

$$5x^2 + x - 2 \equiv A + B(x+1) + Cx(x+1)$$

QUESTION 4 **15 marks** Start a NEW answer booklet **Marks**

(a) For the parabola $x^2 = 12y$ find:

(i) the co-ordinates of the focus **1**

(ii) the equation of the directrix. **1**

(b) A parabola has equation $x^2 = 8(2 - y)$.

(i) Find the coordinates of its vertex**.** **1**

(ii) Find the coordinates of its focus**.** **1**

(iii) Find the equation of its directrix**.** **1**

(iv) Find the *x* and *y* intercepts of the parabola. **2**

(v) Draw a neat sketch of the parabola, illustrating the above information. **1**

(c) Find the vertex of the parabola $y = x^2 + 6x + 7$.

2

(d) Let A and *B* be the fixed points $(-2, 0)$ and $(1, 0)$. Let *P* be the variable point (x, y).

(i) Suppose that *P* moves so that $PA = 2PB$.
Deduce that *P* moves on the circle $x^2 - 4x + y^2 = 0$.

3

(ii) Find the centre and radius of this circle.

2

QUESTION 5 **15 marks** Start a NEW answer booklet **Marks**

(a) For the sequence 1, 1, 2, 3, 5,…

i) Find the next term **1**

ii) Determine whether the sequence is arithmetic, geometric or neither geometric or arithmetic giving reasons. **2**

(b) Find the sum to twenty terms for the following series: **2**

$$80 + 73 + 66 + 59 + \ldots$$

(c) The third term of a geometric sequence is $\frac{4}{27}$ and the fifth term is $\frac{16}{243}$

Find:

(i) the common ratio, **2**

(ii) the first term, **1**

(iii) the limiting sum. **1**

QUESTION 5 continues on the next page

QUESTION 5 (continued) **Marks**

(d) A \$5000 scholarship fund, for tertiary studies, is set up for Johnny at his birth. The account has an interest rate of 8% p.a. compounded quarterly for the duration of the investment.
Johnny commences University on his 18^{th} birthday. An allowance of \$1500 is to be paid to Johnny on commencement of his tertiary studies and each subsequent 3 monthly period.

(i) Show that the amount in the fund just prior to the first allowance payment is \$20 806, to the nearest dollar. **1**

(ii) Show that the balance of the account on Johnny's 19th birthday (ie after the fifth allowance payment) is: **2**

$$5000(1.02)^{76} - 75000(1.02^5 - 1)$$

(iii) Show that the account balance after n payments is given by the expression: **1**

$$5000(1.02)^{71+n} - 75000(1.02)^n + 75000$$

(iv) How long, in years and months, will Johnny receive allowance payments? **2**

QUESTION 6 **15 marks** Start a NEW answer booklet **Marks**

(a) Use differentiation to find the values of x for which the graph of $y = 9x(x + 2)^2$ is:

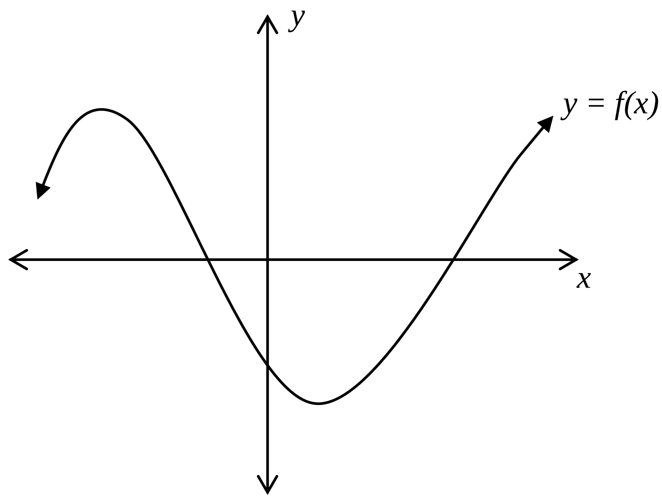
(i) increasing **3**

(ii) concave down. **2**

(b) The gradient function of a curve is $\frac{dy}{dx} = 3x^2 - 2x + 1$. **2**

If the curve passes through the point (2, 3), find its equation.

(c) The diagram shows the graph of a certain function $y = f(x)$. **2**
Copy this graph, then on the same set of axes, draw a sketch of the derivative $y = f'(x)$ of the function.



(d) Consider the function $y = x^3 - 6x^2 + 9x - 4$.

(i) Find its stationary points and determine their nature. **3**

(ii) Find its point of inflexion. **2**

(iii) Graph the function on a number plane. **1**

Year 12 Mathematics Half Yearly Examination 2006

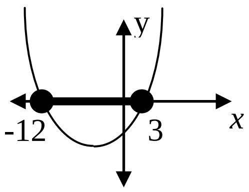
Question No. 1 Solutions and Marking Guidelines

Outcomes Addressed in this Question

P2 provides reasoning to support conclusions which are appropriate to the context

P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities.

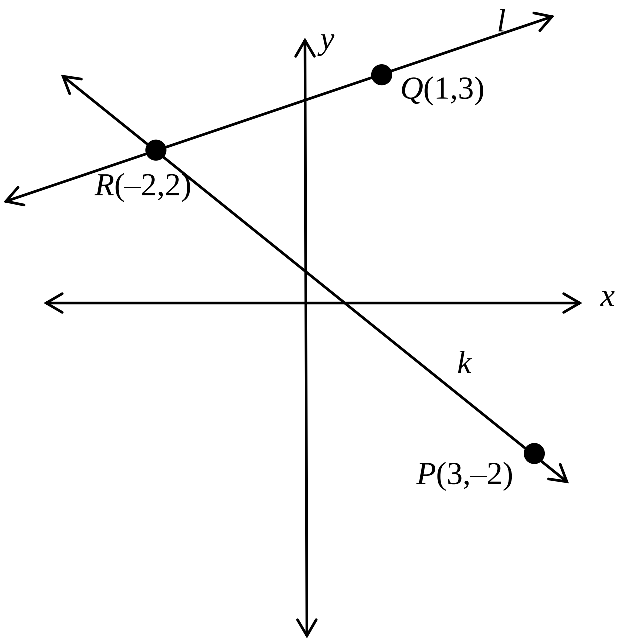
P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques.

Outcome	Solutions	Marking Guidelines
P3	**(a)** $\dfrac{\sqrt{(2.3)^5 - 6.95}}{\pi} = 2.411886541$ $= 2.41$	**2 marks** Correct answer correctly rounded. **1 mark** Correct answer incorrectly rounded **OR** Incorrect answer rounded to 3 sig. figs.
P4	**(b)** $p^2 = 2p$ $p^2 - 2p = 0$ $p(p - 2) = 0$ $p = 0 , 2$	**2 marks** Both correct answers stated. **1 mark** Only 1 of 2 correct answers stated **OR** solution showing correct factorisation leading to incorrect answer.
P4	**(c)** $5x^3 + 40 = 5(x^3 + 8)$ $= 5(x + 2)(x^2 - 2x + 4)$	**2 marks** Expression fully and correctly factorised. **1 mark** Common factor recognised without proceeding to factorise sum of two cubes or factorising difference of two cubes incorrectly.
P2	**(d)** By definition, $\lvert 5t - 32 \rvert$ must be positive. Hence, there are no values of t which will give this expression a value of –18.	**1 mark** Valid explanation given.
P4	**(e)** $x^2 + y^2 = 15 \quad \ldots 1$ $y = 2x \quad \ldots 2$ Sub. eqn. 2 into eqn. 1 $x^2 + (2x)^2 = 15$ $5x^2 = 15$ $x^2 = 3$ $x = \pm\sqrt{3}$	**3 marks** Correct solution **2 marks** Correct solution not showing <u>both</u> values of *x*, **OR** Minor error in solution leading to incorrect answer but working substantially correct. **1 mark** Substantially correct attempt to solve equations simultaneously, not proceeding beyond this point.
P4	**(f)** (i) $36 - 9x - x^2 \geq 0$ $x^2 + 9x - 36 \leq 0$ $(x + 12)(x - 3) \leq 0$  **Solution:** $-12 \leq x \leq 3$	**2 marks** Solution correctly stated. **1 mark** Factorises quadratic correctly, but does not proceed to correct solution. **OR** error leads to an incorrect solution with working consistent with given solution.
P4	(ii) 	**1 mark** Correct representation of solution in part (i), provided incorrect solution in (i) does not lead to simpler graph.
P4	**(g)** $7^m = 18$ $\log 7^m = \log 18$ $m \log 7 = \log 18$ $m = \dfrac{\log 18}{\log 7}$ $= 1.49 \ (2\, dec.\, pl.)$	**2 marks** Correct solution given. **1 mark** Attempts to solve equation using logarithms, showing knowledge of at least one relevant log law.

Year 12 Mathematics Half Yearly Examination 2006
Question No. 2 Solutions and Marking Guidelines

Outcomes Addressed in this Question

P4 chooses and applies appropriate arithmetic, algebraic, graphical and geometric techniques.
H9 communicates using mathematical language, notation diagrams and graphs

Outcome	Solutions	Marking Guidelines
P4 (a)	Substitute (3,–1) in $4x+5y-2=0$ $4\times 3+5\times(-2)-2$ $=12-10-2$ $=0$ $\therefore$(3,–2) lies on k	1 mark correct answer
P4 (b)	$y-y_1=m(x-x_1)$ $y-3=\frac{1}{3}(x-1)$ $3y-9=x-1$ $x-3y+8=0$	2 mark for correct answer. or 1mark correctly using an appropriate form of the straight line
P4 (c)	$x-3y+8=0$ ______(1) $4x+5y-2=0$ ______(2) $4\times(1)$ $4x-12y+32=0$ ______(3) $(2)-(3)$ $17y-34=0$ $y=2$ Sub in (1) $x-6+8=0$ $x=-2$ $\therefore R$(–2,2) is the point of intersection	3 marks for correct answer. or 1 mark solving simultaneously to find *x* mark for attempting to find *y*
H9 (d)		2 marks for correct answer. Only one mark if not labelled correctly.

P4 (e)	$d = \dfrac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2 + b^2}}$ $= \dfrac{3 - 3\times(-2) + 8}{\sqrt{1^2 + 3^2}}$ $= \dfrac{17}{\sqrt{10}}$	2 marks for correct answer. or 1 mark for partially correct answer.
P4 (f)	Length of QR $d = \sqrt{3^2 + 1^2}$ $= \sqrt{10}$ Area of triangle $A = \frac{1}{2}bh$ $= \frac{1}{2}\times QR \times \text{distance of } P \text{ from } l$ $= \frac{1}{2}\times\sqrt{10}\times\dfrac{17}{\sqrt{10}}$ $\text{Area} = \dfrac{17}{2}u^2$	3marks for correct answer. or 1 mark for finding a length of a side and 1 mark for substituting appropriate values in the area formula
P4 (g)	Circle centre (3,–2) and radius $\dfrac{17}{\sqrt{10}}$ $(x-3)^2 + (y+2)^2 = \dfrac{17^2}{10}$	1 mark for stating radius 1 mark for correctly substituting in the central form of a circle

Year 12	Mathematics	Half-Yearly Exam 2006
Question No. 3	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

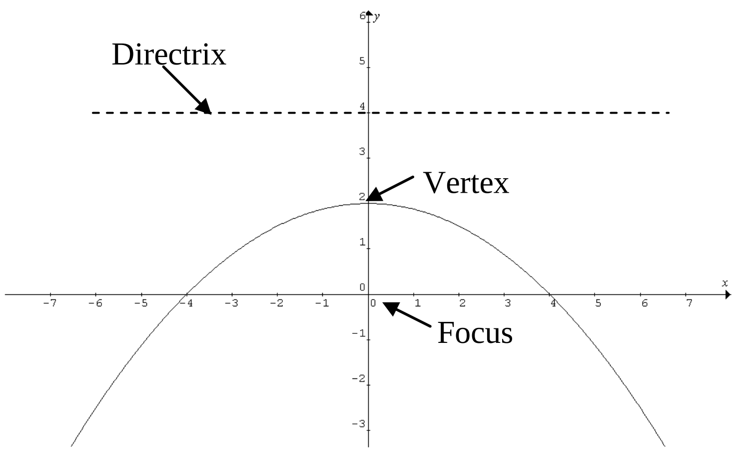
P4 – Chooses and applies appropriate arithmetic and algebraic techniques

Outcome	Solutions	Marking Guidelines
P4	**<u>Question 3</u>** (a) Using quad formula $x = \frac{6 \pm \sqrt{36 - 4 \times 3 \times 1}}{6}$ $= \frac{6 \pm \sqrt{24}}{6}$ $=1.82$ or 0.18	Award 1 for correct substitution into correct formula Award 2 for correct substitution and at least one correctly evaluated answer
	(b) (i) $\alpha + \beta = -\frac{b}{a}$ $= 5$	Award 1 for correct answer.
	(ii) $\alpha\beta = \frac{c}{a}$ $= 2$	Award 1 for correct answer.
	(iii) $(\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1$ $= 2+5+1$ $= 8$	Award 1 for simplification Award 2 for simplification and correct substitution (subsequent errors are not penalised)
	(c) $\Delta = b^2 - 4ac$ $= (k+2)^2 - 4 \times 1 \times 4$ $= k^2 + 4k - 12$ $= (k+6)(k-2)$	Award 1 for correct calculation of Δ.
	(i) Equal roots $\Delta = 0$ $\therefore (k+6)(k-2) = 0$ $\therefore k = 2$ *or* $k = -6$	Award 1 for correct answer
	(ii) Distinct real roots $\Delta > 0$ $\therefore (k+6)(k-2) > 0$ $\therefore k > 2$ or $k < -6$	Award 1 for stating $\Delta > 0$ (Deduct mark for $\Delta \geq 0$)
	(d) Let $3^x = u$ $\therefore 3^{x \times 2} + 2 \times 3^x - 15 = 0$ becomes $u^2 + 2u - 15 = 0$ $\therefore (u+5)(u-3) = 0$ $\therefore 3^x = -5$ *or* $3^x = 3$ no solutions $x = 1$ Ie the only solution is $x = 1$	Award 1 for correct substitution Award 2 for correct solutions for "u" Award 3 for both correct solutions for x. (ie. some indication that student considered $3^x = -5$, but decided that this would yield NO solution)

	(e) $5x^2 + x - 2 \equiv A + B(x+1) + Cx(x+1)$ $= A + Bx + B + Cx^2 + Cx$ $= Cx^2 + (B+C)x + (A+B)$ $\therefore\ C = 5 \qquad B + C = 1 \qquad A + B = -2$ $B + 5 = 1 \qquad A - 4 = -2$ $\therefore B = -4 \qquad A = 2$ **OR** By substitution : Let $x = -1$ $5(-1)^2 + (-1) - 2 = A$ $\therefore\ A = 2$ Let $x = 0$ $\therefore\ -2 = A + B$ $= 2 + B$ $\therefore\ B = -4$	Award 1 for correct expansion Award 2 for correct expansion and collecting like terms Award 3 for at least one correct solution resulting from equating coefficients Award 1 each for each correct substitution and at least one correct solution

Year 12	Mathematics	Half Yearly Examination 2006
Question No.4	Solutions and Marking Guidelines	

Outcome Addressed in this Question		
H2	constructs arguments to prove and justify results	
H9	communicates using mathematical language, notation, diagrams and graphs	
Outcome	**Solutions**	**Marking Guidelines**

(a) (i) H9	$x^2 = 12y = 4.3.y$ $\therefore a = 3$ $\therefore$ Focus = $(0,3)$	**Award 1** for correct answer
(ii) H9	The directrix is $y = -3$	**Award 1** for correct answer
(b) (i) H9	$x^2 = 8(2-y) = -8(y-2) = -4.2.(y-2)$ $\therefore a = 2$ Vertex = $(0,2)$	**Award 1** for correct answer
(ii) H9	Focus = $(0,0)$	**Award 1** for correct answer
(iii) H9	The directrix is $y = 4$	**Award 1** for correct answer
(iv) H9	When $y = 0$, $x^2 = 8(2-0)$ $\therefore x^2 = 16$ $\therefore x = \pm 4$ The x intercepts are $(4,0)$ and $(-4,0)$. When $y = 0$, $0^2 = 8(y-2)$ $\therefore y = 2$ The y intercept is $(0,2)$.	**Award 2** for correct intercepts **Award 1** for correct x – intercept *or* **Award 1** for correct y – intercept
(v) H9	 	**Award 1** for correct answer

(c) H9	$y = x^2 + 6x + 7$ $y' = 2x + 6$ Vertex occurs where $y' = 0$ $\therefore 2x + 6 = 0$ $\therefore x = -3$ $\therefore$ Vertex is $(-3, -2)$	**Award 2** for correct solution **Award 1** for substantial progress towards solution
(d) (i) H2	$PA = 2PB$ $\sqrt{(x+2)^2 + (y-0)^2} = 2 \times \sqrt{(x-1)^2 + (y-0)^2}$ $\therefore (x+2)^2 + y^2 = 4 \times \left[(x-1)^2 + y^2\right]$ $\therefore x^2 + 4x + 4 + y^2 = 4 \times \left[x^2 - 2x + 1 + y^2\right]$ $\therefore x^2 + 4x + 4 + y^2 = 4x^2 - 8x + 4 + 4y^2$ $\therefore 3x^2 - 12x + 3y^2 = 0$ $\therefore x^2 - 4x + y^2 = 0$	**Award 3** for correct solution **Award 2** for substantial progress towards solution **Award 1** for either: removing square root *or* expansion
(ii) H2	$x^2 - 4x + y^2 = 0$ $x^2 - 4x + 4 + y^2 = 4$ $(x-2)^2 + y^2 = 4$ $\therefore$ Centre = $(2, 0)$ $\therefore$ Radius = 2	**Award 2** for correct solution **Award 1** for either: centre *or* radius

Year 12 Question No. 5 Guidelines	Mathematics Solutions and Marking	Task 2Examination 2006

Outcomes Addressed in this Question

H2 constructs arguments to prove and justify results

H4 expresses practical problems in mathematical terms based on simple given models

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems

Outcome	Solutions	Marking Guidelines
	a **1, 1, 2, 3, 5,…….**	
H5	i) next term: 3+5 = 8	1 mark correct term
H2	ii) Arithmetic if $T_1 - T_2 = T_3 - T_2$ $1 - 1 \neq 2 - 1$ $\therefore$ not arithmetic Geometric if $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ $\frac{2}{1} \neq \frac{1}{1}$ $\therefore$ not geometric $\therefore$ Series is neither arithmetic or geometric	1 mark correct conclusion 1 mark for reasons
H5	**b** $80 + 73 + 66 + 59 + ...$ $a = 80, \quad d = -7, \quad n = 20$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $S_{20} = \frac{20}{2}\left[2\times 80 + (20-1)\times -7\right]$ $S_{20} = 270$	2 marks correct solution 1 mark substantially correct
H5	**c** *i)* $ar^2 = \frac{2}{27}$ $ar^4 = \frac{16}{243}$ $r^2 = \frac{16}{243} \times \frac{27}{4}$ $r^2 = \frac{4}{9}$ $r = \pm\frac{2}{3}$	2 marks correct method giving both solutions for *r* 1 mark 1 correct solution for *r*
	ii) $a\left(\pm\frac{2}{3}\right)^2 = \frac{4}{27}$ $a = \frac{1}{3}$	1 mark correct solution
	iii) $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{\frac{1}{3}}{1-\frac{2}{3}} \quad or \quad \frac{\frac{1}{3}}{1-\frac{-2}{3}}$ $S_\infty = 1 \quad or \quad \frac{1}{5}$	1 mark correct solution

	d	
	i) $5000(1+0\cdot 02)^{72}$	1 mark correct answer
	\$20806	
H5	*ii*) $A_1 = 5000(1\cdot 02)^{72} - 1500$	
	$A_2 = [5000(1\cdot 02)^{72} - 1500]\times 1\cdot 02 - 1500$	2 marks correct method
H4	$A_3 = \left\{[5000(1\cdot 02)^{72} - 1500]\times 1\cdot 02 - 1500\right\}\times 1\cdot 02 -$	1 mark substantially correct
	$\therefore \quad A_5 = 5000(1\cdot 02)^{76} - 1500(1 + 1\cdot 02 + ... + 1\cdot 02^4)$	
	$S_n(GP) = \dfrac{a(r^n - 1)}{r - 1} = \dfrac{1(1\cdot 02^5 - 1)}{1\cdot 02 - 1}$	
	$S_5 = 50(1\cdot 02^5 - 1)$	
	$\therefore \quad A_5 = 5000\times 1\cdot 02^{76} - 75000\left(1\cdot 02^5 - 1\right)$	
	iii) $A_5 = 5000\times 1\cdot 02^{71+5} - 75000\left(1\cdot 02^5 - 1\right)$	1 mark correct method
	$\therefore \quad A_n = 5000\times 1\cdot 02^{71+n} - 75000\left(1\cdot 02^n - 1\right)$	
	$A_n = 5000\times 1\cdot 02^{71+n} - 75000\times 1\cdot 02^n + 75000$	
	iv) Allowance finished when $A_n = 0$	
	$0 = 5000\times 1\cdot 02^{71+n} - 75000\times 1\cdot 02^n + 75000$	1 mark giving $A_n = 0$
	$75000\times 1\cdot 02^n - 5000\times 1\cdot 02^{71+n} = 75000$	1 mark correct method to gain 16 repayments
	$1\cdot 02^n\left(75000 - 5000\times 1\cdot 02^{71}\right) = 75000$	
	$1\cdot 02^n = \dfrac{75000}{75000 - 5000\times 1\cdot 02^{71}}$	
	$n = \dfrac{\log\left(\dfrac{75000}{75000 - 5000\times 1\cdot 02^{71}}\right)}{\log\left(1\cdot 02\right)}$	
	$n \approx 16\cdot 03...$	
	$n = 16$	
	ie. There are 16 payments and 3 years 9 months.	

Year 12 Mathematics Half-Yearly Examination 2006
Question 6 solutions and marking guidelines

Outcomes addressed in this question

P5 understands the concept of a function and the relationship between a function and its graph
P6 relates the derivative of a function to the slope of its graph
P7 determines the derivative of a function through routine application of the rules of differentiation
P8 understands and uses the language and notation of calculus
H5 applies appropriate techniques from the study of calculus to solve problems
H6 uses the derivative to determine the features of the graph of a function
H7 uses the features of a graph to deduce information about the derivative

Solution (✓ = 1 mark)

(a) (i) $y = 9x(x+2)^2$

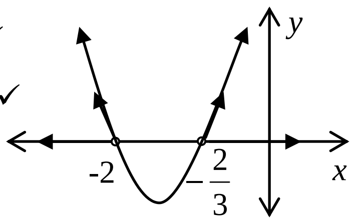
$u = 9x \qquad v = (x+2)^2$
$u' = 9 \qquad v' = 2(x+2)$

$y' = 9x.2(x+2) + 9(x+2)^2$
$\quad = 18x(x+2) + 9(x^2 + 4x + 4)$
$\quad = 27x^2 + 72x + 36$ ✓

y' is increasing when $y' > 0$.

$27x^2 + 72x + 36 > 0$
$3x^2 + 8x + 4 > 0$
$(3x+2)(x+2) > 0$ ✓

$x < -2$ or $x > -\frac{2}{3}$ ✓



(ii) $y'' = 54x + 72$ ✓

Concave down when $y'' < 0$

$54x + 72 < 0$

$x < -\frac{72}{54}$

$x < -1\frac{1}{3}$ ✓

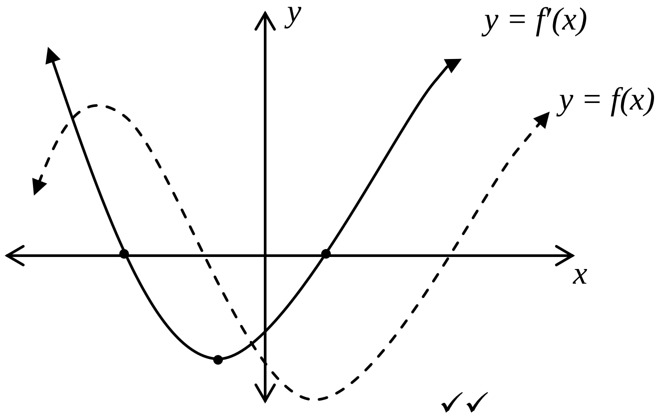
(b) $y' = 3x^2 - 2x + 1$

$y = x^3 - x^2 + x + c$ ✓

When $x = 2$, $y = 3$: $\quad 3 = 2^3 - 2^2 + 2 + c$
$c = -3$ ✓

$\therefore\ y = x^3 - x^2 + x - 3$

(c)



✓✓

- where $f(x)$ is a stationary point, $f'(x) = 0$ (x-intercepts)
- where $f(x)$ is a maximum point, $f'(x)$ changes from positive to negative

Outcome, marking guidelines, examiner's comment

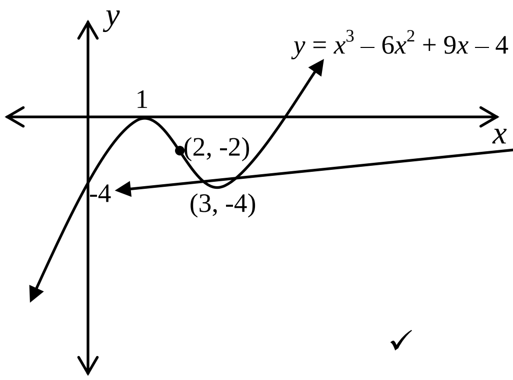
In Q6, responses ranged from students who were careful, precise and knew exactly what they were doing to students with a limited or careless understanding of basic calculus concepts.

P5, P6, P7, P8, H5, H6: 3 marks
3 marks: correct inequality found
2 marks: quadratic expression factorised
1 mark: derivative found
Students made most errors here, with many not differentiating correctly. Some did not understand the theory behind increasing curves ($y' > 0$) and found stationary points instead. Sloppy algebra led to long incorrect solutions. If the algebra looks wrong and complicated, check it!

P5, P6, P7, P8, H5, H6: 2 marks
2 marks: correct inequality found
1 mark: second derivative found
Again, some students did not understand the theory behind 'concavity'. Careless errors included forgetting the '-' sign in the answer or not simplifying the fraction.

P6, H5, H6: 2 marks
2 marks: primitive with c evaluated
1 mark: primitive with c not evaluated
Quite well done, though some students did not understand this question and found the equation of a tangent instead.

P6, H5, H6, H7: 2 marks
2 marks: correct graph
1 mark: any graph with correct x-intercepts
Very well done!

• where $f(x)$ is a minimum point, $f'(x)$ changes from negative to positive • where $f(x)$ is a point of inflexion, $f''(x) = 0$ so $f'(x)$ has a stationary point	
(d) (i) $y = x^3 - 6x^2 + 9x - 4$ $y' = 3x^2 - 12x + 9$ $y'' = 6x - 12$ Stationary points where $y' = 0$ $3x^2 - 12x + 9 = 0$ $x^2 - 4x + 3 = 0$ $(x - 3)(x - 1) = 0$ $x = 1$ or 3. ✓ When $x = 1$, $y = 1 - 6 + 9 - 4 = 0$ (1, 0) $y'' = 6 - 12 = -6 < 0$ $\therefore$ (1, 0) is a maximum point. ✓ When $x = 3$, $y = 27 - 54 + 27 - 4 = -4$ (3, -4) $y'' = 18 - 12 = 6 > 0$ $\therefore$ (3, -4) is a minimum point. ✓	**P5, P6, P7, P8, H5, H6: 3 marks** 3 marks: both stationary points determined 2 marks: one stationary point determined or both stationary points found but not determined 1 mark: x-coordinates of stationary points Generally well done: students know the process well.
(ii) Point of inflexion where $y'' = 0$. $6x - 12 = 0$ $x = 2$. ✓ <table><tr><td>x</td><td>1</td><td>2</td><td>3</td></tr><tr><td>y"</td><td>-6</td><td>0</td><td>6</td></tr></table>⌒ ⌣ Concavity changes. When $x = 2$, $y = 8 - 24 + 18 - 4 = -2$. (2, -2) is the point of inflexion. ✓	**H5, H6: 2 marks** 2 marks: point of inflexion found and tested 1 mark: x-coordinate found only Many students lost a mark for not testing the point for change in concavity.
(iii) y-intercept is -4.  ✓	**H5, H6: 1 mark** 1 mark: graph shown with y-intercept and both stationary points marked Many students did not get the mark because they did not show the y-intercept, which was easy to find. Some Extension 1 students used polynomial division to find the other x-intercept: this was not necessary here.